What makes the Quantum Computer different?

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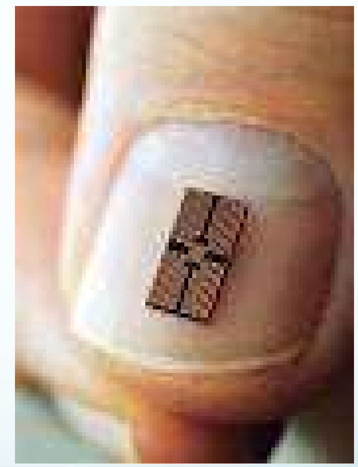
Basics of Computation

Prepare → **Process** → **Read out**

- **Three stages of computation:**
  - **Preparation of Information:** convert data into 0s and 1s,
  - **Processing of Information:** calculations of interest,
  - **Read-out of results of the calculations:** get the answers.
- Done on billions of transistors that sit on a chip
- These transistors obey the laws of the classical world
- What if they obey the laws of quantum world instead?

Source: The internet



$|\Psi\rangle$

Source: The internet

Come plug yourself into the world of quantum mechanics

Quantum mysteries

- What makes the quantum computer different?
  - ✓ **Wave-particle duality,**
  - ✓ **the Wavefunction Ψ as a linear superposition,**
  - ✓ **Quantum interference phenomena, &**
  - ✓ **Quantum Entanglement**

Source: The internet

Our progress with the creation of quantum computers needs us to stand on the shoulders of these giants.

Newton’s Universe

The quantities we measure in the world around us (the “classical” world) are all continuous.

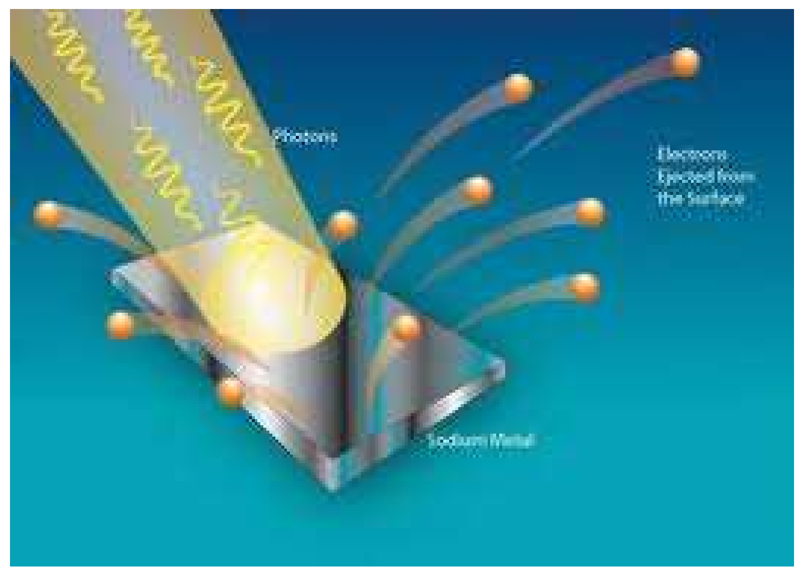
Newton

Energy (E), momentum (p), position (x) ….
they take any value they want: integer, fraction, irrational

- ✓ **But is this also true of the “quantum” world ?**

- ✓ **Is light a wave or a particle ? Some experiments suggest that it’s a wave, but some others ….**

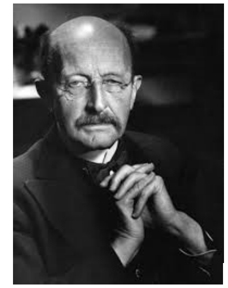
Photoelectric Effect



Source: The internet

Quantisation of physical quantities

Planck



Explained Blackbody Radiation with this idea!

1918

✓ **Energy of wave-like light comes in multiples of some quantity (i.e. it is quantised into packets)**

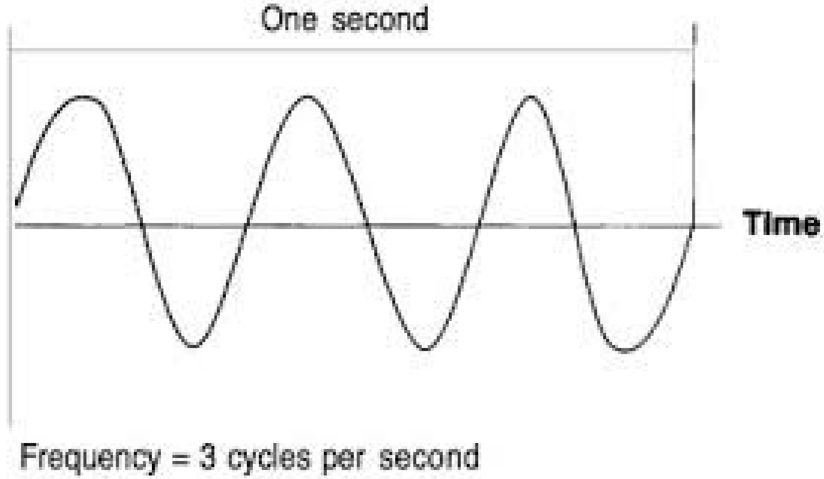
Frequency of light

Photon Energy

$$\mathrm{E} = nh\nu \quad (n \in \mathcal{Z})$$

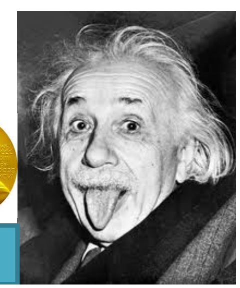
Planck's constant

Frequency = no. of cycles of wave per second



Source: The internet

Einstein



1921

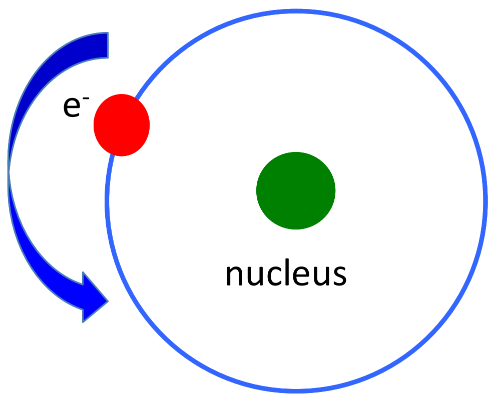
Explained Photoelectric Effect with this idea!

Quantisation of physical quantities

✓ **Stability of the atom: electrons forms stable orbits around nucleus as their angular momentum comes in multiples of some quantity (i.e. it is quantised into packets)**

Angular momentum of e^- orbit

$$p_\phi = n\,h/2\pi \qquad (n \in \mathcal{Z})$$



n = no. of times
e- goes around
circle in 1 second

Bohr

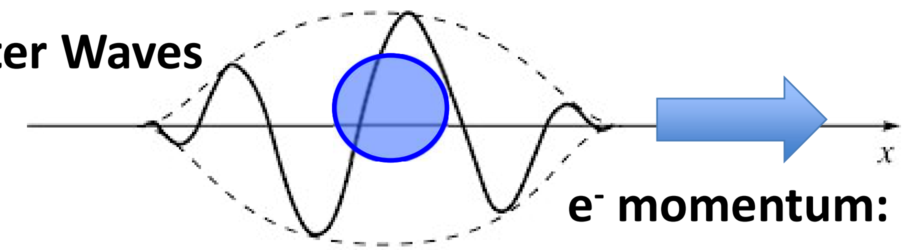
Wave-particle duality

de Broglie

1929

de Broglie hypothesised: the particles that make up matter have wave-like properties too!

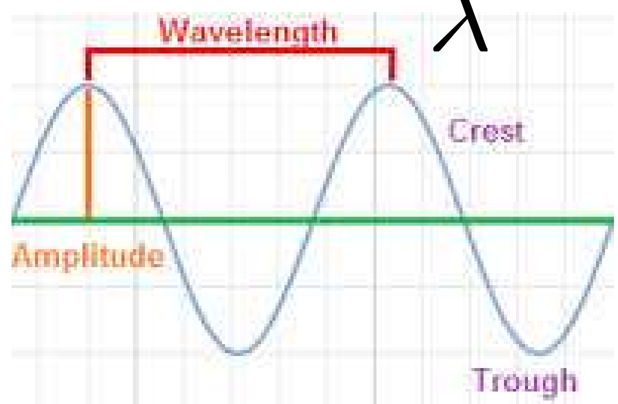
Matter Waves



e⁻ momentum: p

$$p = \frac{h}{\lambda}$$

λ

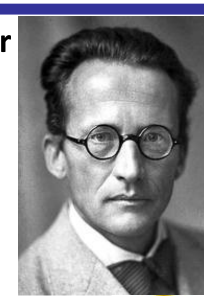


The Wavefunction

Bohr

1922

Schroedinger



1933

All relevant information of the quantum system is in its wavefunction : $\Psi \in \mathcal{C}$

But this description of nature is probabilistic : the probability that a quantum particle has some particular property : $P = |\Psi|^2$

Dirac

1933

1954

Born

Schroedinger and Dirac found equations that described how matter-waves move through space as time evolves :

Quantum Mechanics is born !

Quantum Uncertainty, Exclusion & Symmetry

Quantum mechanical objects cannot simultaneously have precisely determined position (x) and momentum (p)

$$\Delta p \, \Delta x \;\geq\; h/2\pi$$

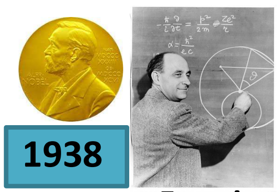
1932

Heisenberg

Δp : Uncertainty in momentum

Δx : Uncertainty in position

S.N.Bose

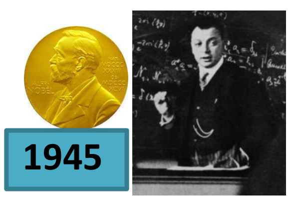


1938

Fermi

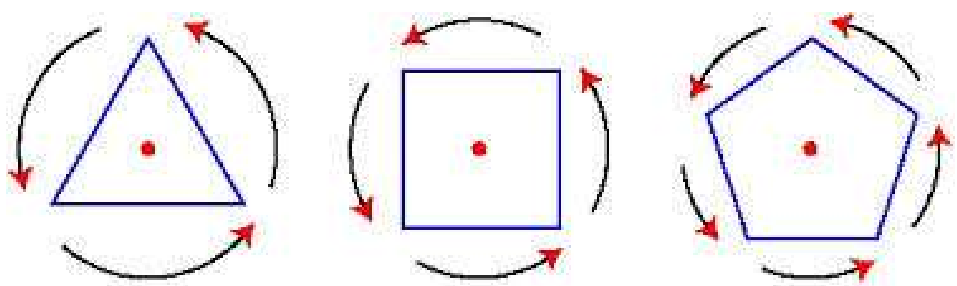
Nature has 2 types of particles : fermions (matter) & bosons (force).
No two Fermions can have identical properties (Pauli Exclusion Principle).

Symmetries, and what Nature does with them, are key concepts.



1945

Pauli



3-Fold Symmetry   4-Fold Symmetry   5-Fold Symmetry

1963

Wigner

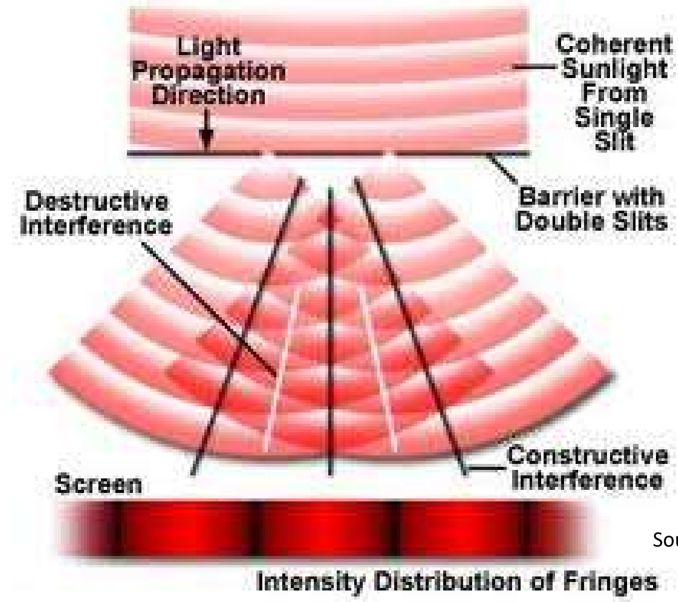
Double-slit interference with light

Interference of Water Waves

Source: The internet

What happens when two stones are thrown simultaneously into water ? The pretty patterns made by the ripples as they meet is called an interference pattern.

Interference of Light Waves



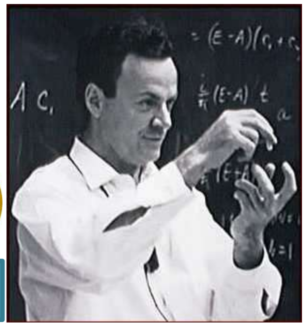
Source: The internet

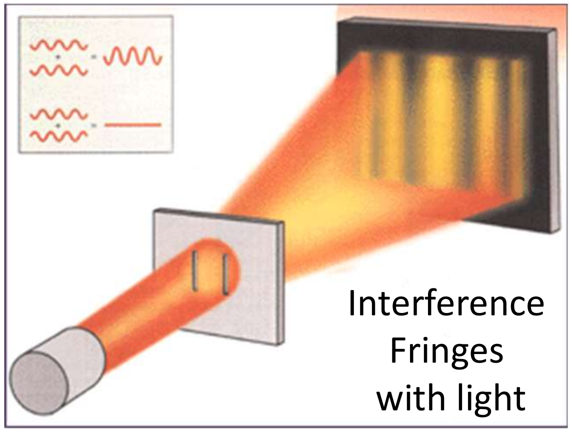
Thomas Young's famous experiment (1803)

Double-slit interference with electrons

Feynman

1965

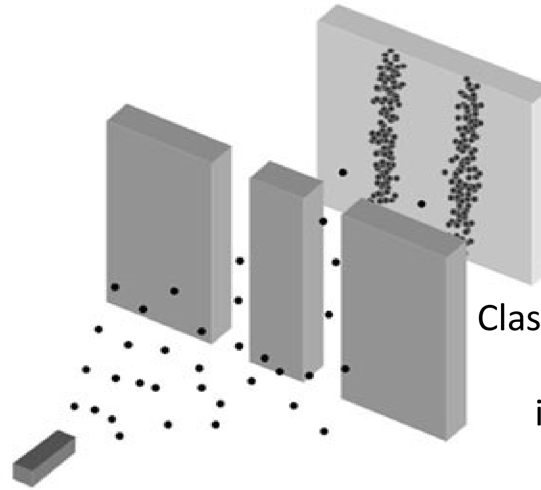




Interference Fringes with light

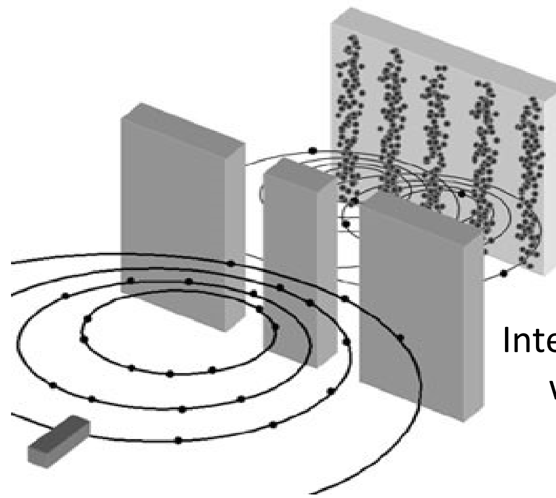
Source: The internet

Hrvoje Crvelin



Classical “Heaping up” of particles into two fringes

What would an interference experiment with electrons show ?



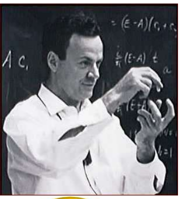
Interference Fringes with electrons

The same as light !
WHY?!

A. Tonomura (1989)

Double-slit interference with electrons

Feynman



1965

What would an interference experiment with electrons show ?

The same as light!

Probability Distribution $P = |\psi|^2 \; , \; \psi \in \mathcal{C}$

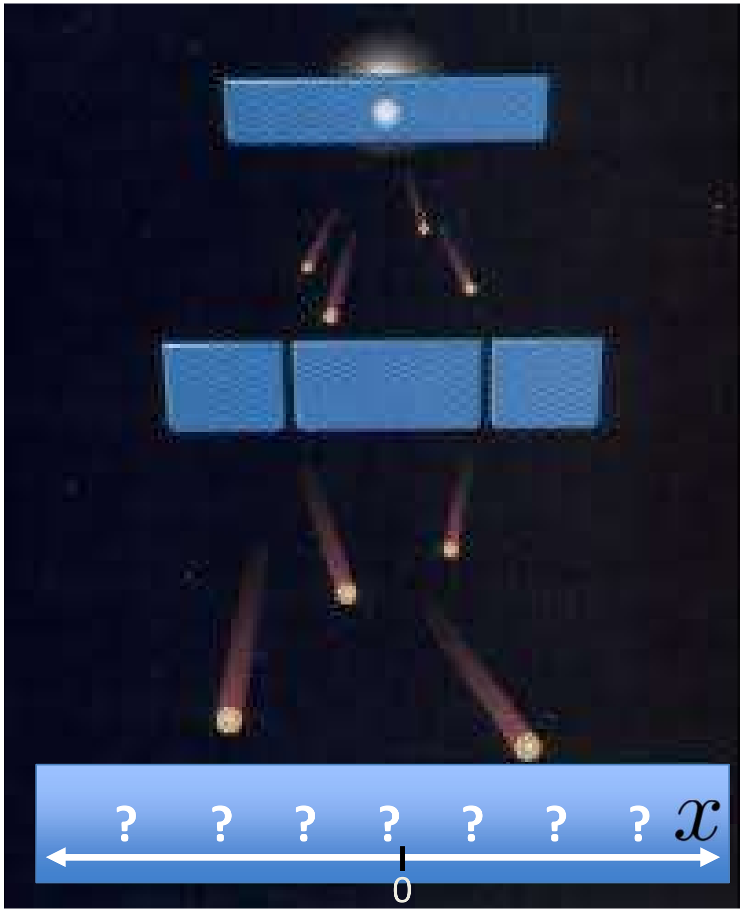
Linear superposition $\psi_{2-\text{slit}} = \psi_{\text{slit }1} + \psi_{\text{slit }2}$

$(\psi_1 = |\psi_1| e^{i\delta_1(x)} \text{ and } \psi_2 = |\psi_2| e^{i\delta_2(x)})$

Tonomura (1989)

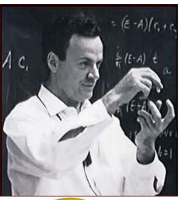
Source: The internet

Interference with Particles ?



Feynman

Double-slit interference with electrons



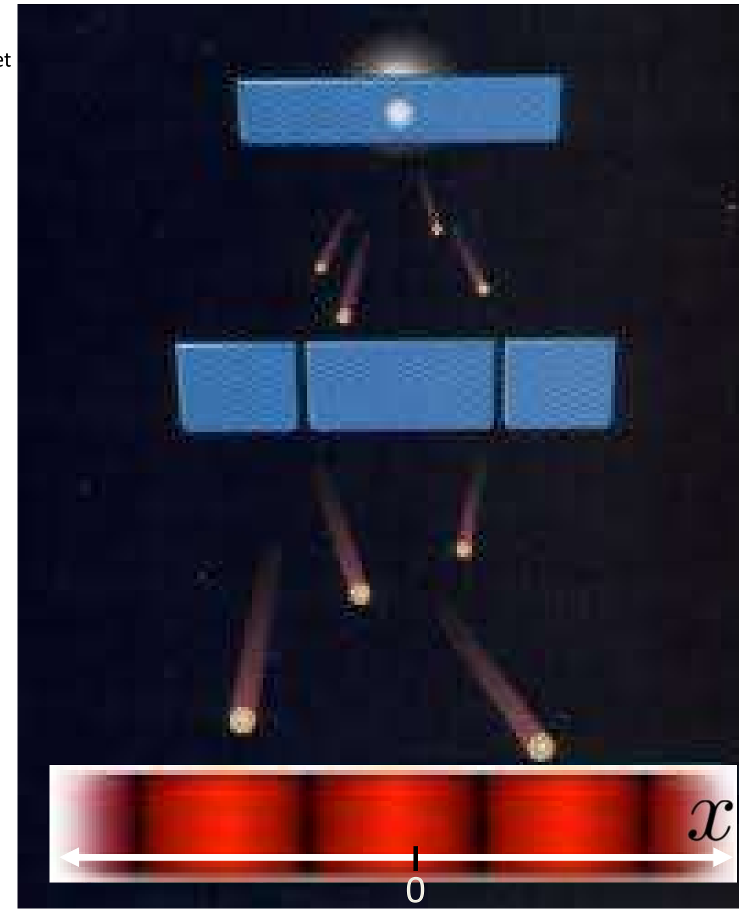
1965

What would an interference experiment with electrons show ?

The same as light!

Source: The internet

Interference with Particles ?



Probability Distribution $P = |\psi|^2 \ , \ \psi \in \mathcal{C}$

Linear superposition $\psi_{2-\mathrm{slit}} = \psi_{\mathrm{slit}\ 1} + \psi_{\mathrm{slit}\ 2}$

$(\psi_1 = |\psi_1| e^{i\delta_1(x)}$ and $\psi_2 = |\psi_2| e^{i\delta_2(x)})$

Quantum Inteference

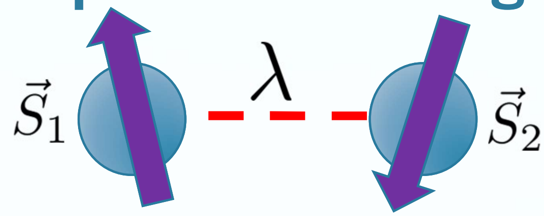
$$\begin{aligned} P &= |\psi_{2-slit}|^2 \\ &= |\psi_1|^2 + |\psi_2|^2 + 2\psi_1^*\psi_2 \\ &= \underbrace{P_1 + P_2}_{\text{Classical Probabilities}} + \underbrace{2\sqrt{P_1 P_2}\cos\delta(x)}_{\text{Quantum Interference}} \end{aligned}$$

$(|\psi_1| = \sqrt{P_1} \ , \ |\psi_2| = \sqrt{P_2}$ and $\delta = \delta_2 - \delta_1)$

Tonomura (1989)

The world in a grain of sand: the 2-spin Heisenberg model

- A lot can be learnt by studying a simple problem.
- Consider two spin-1/2 moments interacting via Hamiltonian



$$H = \lambda \vec{S}_1 \cdot \vec{S}_2$$

Antiferromagnetic Interaction:
$\lambda > 0$

Good Quantum Numbers

$$[H, S^2] = 0, \quad \text{where } S^2 = \left(\vec{S}_1 + \vec{S}_2\right)^2$$

$$[H, S_z] = 0, \quad \text{where } S_z = (S_{1z} + S_{2z})$$

Eigenstates

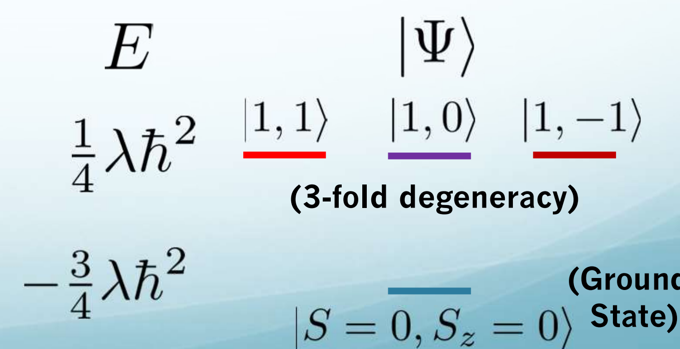
$$|S, m_S\rangle$$

m_S: z-component of combined spin

S: $\sqrt{S(S+1)}\hbar$ Length of combined spin

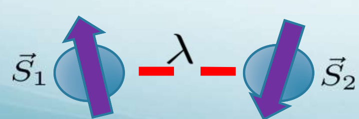
Energy Eigenvalue / **Eigenstate of the system**

E	$\vert\Psi\rangle$
$-\frac{3}{4}\lambda\hbar^2$	$\vert S=0, S_z=0\rangle = \frac{1}{\sqrt{2}}\left(\vert\uparrow,\downarrow\rangle - \vert\downarrow,\uparrow\rangle\right)$
$\frac{1}{4}\lambda\hbar^2$	$\vert S=1, S_z=0\rangle = \frac{1}{\sqrt{2}}\left(\vert\uparrow,\downarrow\rangle + \vert\downarrow,\uparrow\rangle\right)$ $\vert S=1, S_z=1\rangle = \vert\uparrow,\uparrow\rangle$ $\vert S=1, S_z=-1\rangle = \vert\downarrow,\downarrow\rangle$



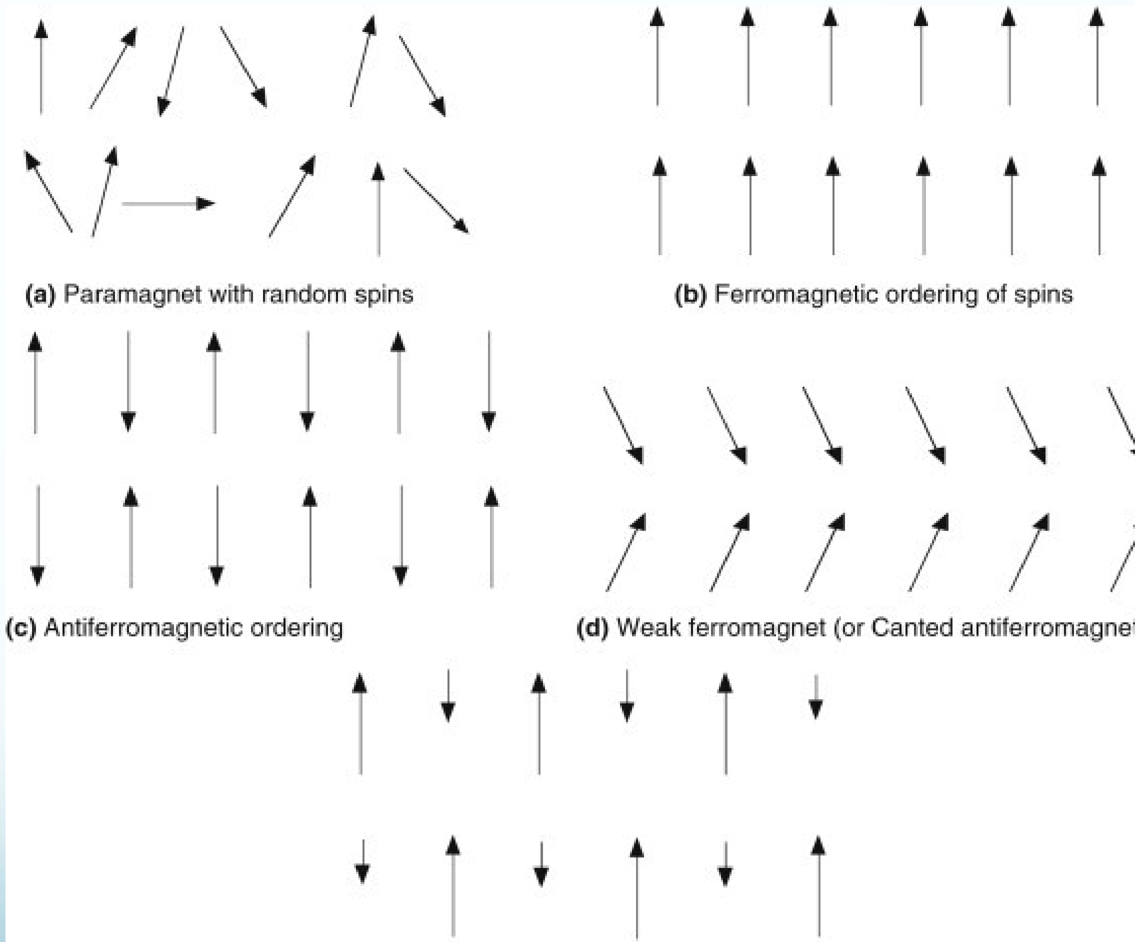
Symmetries & the ground state

- Various forms of magnetism observed in nature are described by ground state (i.e., equilibrium) configurations of magnetic moments (spins) that are easy to guess
- Examples include ferromagnetism, antiferromagnetism etc.
- But the ground state of the two spin system with Heisenberg interaction is different. Why?



$$H = \lambda \vec{S}_1 \cdot \vec{S}_2$$

$$\frac{1}{\sqrt{2}}\left(|\uparrow,\downarrow\rangle - |\downarrow,\uparrow\rangle\right)$$

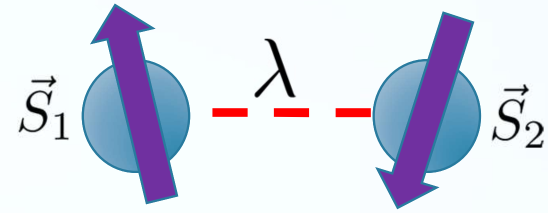


(a) Paramagnet with random spins
(b) Ferromagnetic ordering of spins
(c) Antiferromagnetic ordering
(d) Weak ferromagnet (or Canted antiferromagnet)
(e) Ferrimagnetic ordering

Source: The internet

Symmetries & the ground state

$$H = \lambda \vec{S}_1 \cdot \vec{S}_2$$



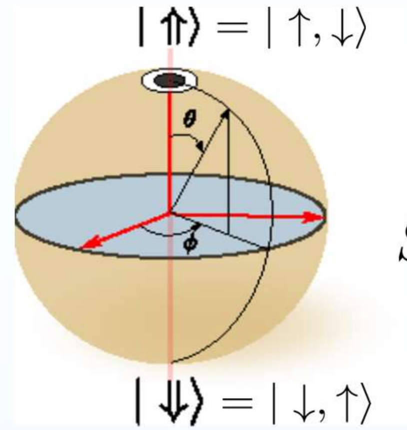
- The ground state is not

$$|\uparrow,\downarrow\rangle \text{ or } |\downarrow,\uparrow\rangle$$

- Instead, it is the **singlet**

$$\frac{1}{\sqrt{2}}\left(|\uparrow,\downarrow\rangle - |\downarrow,\uparrow\rangle\right)$$

- Singlet preserves rotational symmetry of Hamiltonian (on 3D sphere of total spin S)



Bloch sphere of total spin S

$$S^2 = (\vec{S}_1 + \vec{S}_2)^2$$

- The symmetry of the singlet & triplet-0 ($\frac{1}{\sqrt{2}}(|\uparrow,\downarrow\rangle + |\downarrow,\uparrow\rangle)$) states is preserved due to **quantum fluctuations**

- Singlet and Triplet-0 eigenstates possess **entanglement** between the two spins

Not Eigenstates of H

$|\uparrow,\downarrow\rangle$ $|\downarrow,\uparrow\rangle$

Quantum Fluctuations

(action of $S_1^+ S_2^- + S_1^- S_2^+$ terms in H)

Eigenstates of H

$\frac{1}{\sqrt{2}}(|\uparrow,\downarrow\rangle + |\downarrow,\uparrow\rangle)$

$\frac{1}{\sqrt{2}}(|\uparrow,\downarrow\rangle - |\downarrow,\uparrow\rangle)$

Entanglement basics

- **Entanglement** is the property of a composite system
  - with sub-systems interacting with one another in some way
  - whose states cannot be written as separable decompositions of the states of the individual sub-systems
  - state of one spin cannot be given without that of other
  - Measuring angular momentum of one spin will also give that of the other --- the results will be anti-correlated.

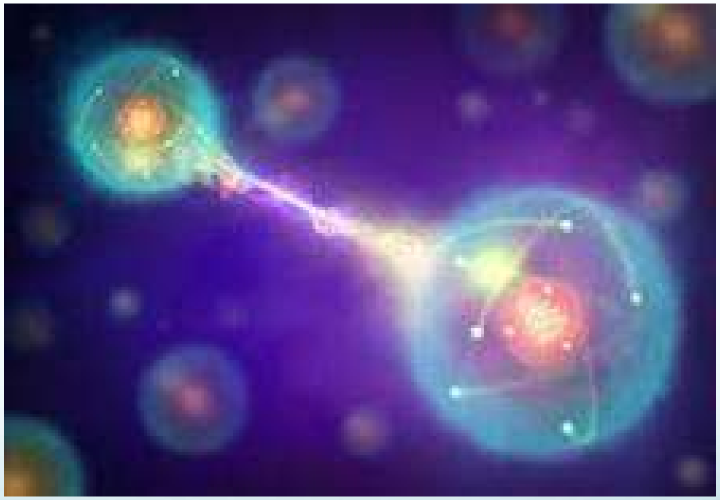
$$\frac{1}{\sqrt{2}}(|\uparrow,\downarrow\rangle - |\downarrow,\uparrow\rangle)$$

Entangled Singlet state: cannot be written as a direct product (or separable) state

$$|\uparrow,\downarrow\rangle \;\&\; |\downarrow,\uparrow\rangle$$

Un-Entangled states: manifestly direct product (or separable) in nature

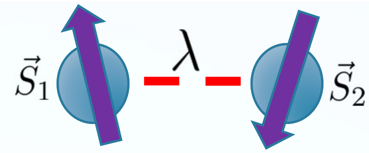
Source: The internet



- Entanglement is a measure of quantum correlations that can be non-local

Computing Entanglement

$$H = \lambda \vec{S}_1 \cdot \vec{S}_2$$




Boltzmann

- Consider the singlet state for two spin-1/2 moments A & B

$$|\psi_{AB}\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow_A\rangle|\downarrow_B\rangle - |\downarrow_A\rangle|\uparrow_B\rangle\right)$$

- The density matrix obtained from this singlet is taking an outer product

$$\rho^{AB} = |\psi_{AB}\rangle\langle\psi_{AB}| = \frac{1}{2}\left(|\uparrow_A\rangle|\downarrow_B\rangle - |\downarrow_A\rangle|\uparrow_B\rangle\right)\left(\langle\uparrow_A|\langle\downarrow_B| - \langle\downarrow_A|\langle\uparrow_B|\right)$$

- Trace over all configurations of B to get the reduced density matrix for A

$$\rho^A = \langle\uparrow_B|\rho_{AB}|\uparrow_B\rangle + \langle\downarrow_B|\rho_{AB}|\downarrow_B\rangle = \frac{1}{2}\left(|\uparrow_A\rangle\langle\uparrow_A| + |\downarrow_A\rangle\langle\downarrow_A|\right) = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$$

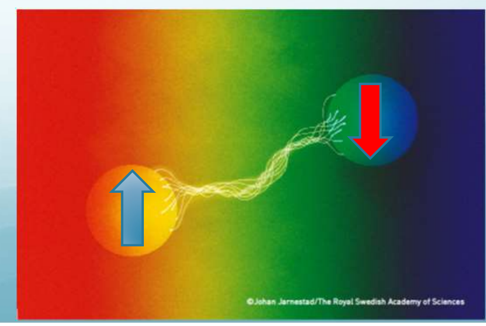
(remember that some terms will disappear as $\langle\uparrow_B|\downarrow_B\rangle = 0 = \langle\downarrow_B|\uparrow_B\rangle$)

- Entanglement entropy for spin A can be computed from ρ^A

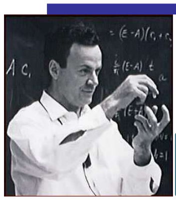
$$S_A = -\sum_i \rho_i^A \ln \rho_i^A = 2 \times \frac{1}{2}\ln 2 = \ln 2 = S_B$$

Maximally Entangled!


Von Neumann



Quantum Computation as simulation of quantum systems



Feynman

1965

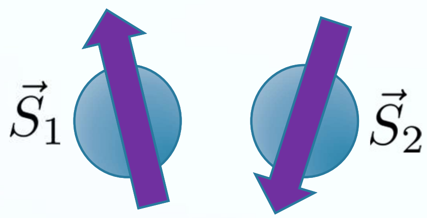
R. P. Feynman. Int. J. Theor. Phys. 21, 467 (1982).
Feyman conjectured that quantum computers will simulate quantum systems more efficiently than classical computers.

- Simulating full time evolution of quantum systems on a classical computer is intractable: the quantum system's Hilbert space dimension grows exponentially with the size of the system.
- Storing the state of 40 spin-1/2 particles in a classical memory requires $2^{40} \approx 10^{12}$ numbers, calculating it's time evolution requires exponentiation of a 2^{40} X 2^{40} matrix with 10^{24} entries!
- It is exponentially difficult problem to store & manipulate the state of a quantum system.
- Can we bypass this headache by using one quantum system to simulate another directly? i.e., states of simulator obey the same equations of motion as states of simulated system.
- Feynman conjectured existence of a class of universal quantum simulators capable of simulating any quantum system that evolves according to local interactions.

(Seth Lloyd, Science, 1996)

Quantum Computation Basics

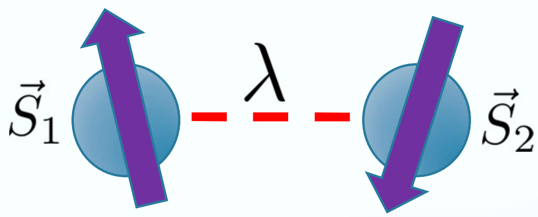
Prepare → **Process** → **Read out**



- Prepare state of two non-interacting spin-1/2s

$$|\Psi\rangle = |S_{1z}\rangle \otimes |S_{2z}\rangle$$

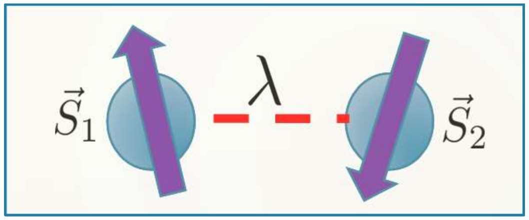
- State is separable/direct product: no entanglement
- Paramagnetic response to external B-field



$$H = \lambda \vec{S}_1 \cdot \vec{S}_2$$

Ferromagnetic Interaction: $\lambda < 0$

- Start interaction via application of quantum logic gates
- Reach ground state $\frac{1}{\sqrt{2}}(|\uparrow,\uparrow\rangle + |\downarrow,\downarrow\rangle)$



$$H = \lambda \vec{S}_1 \cdot \vec{S}_2 + B\left(S_{1z} + S_{2z}\right)$$

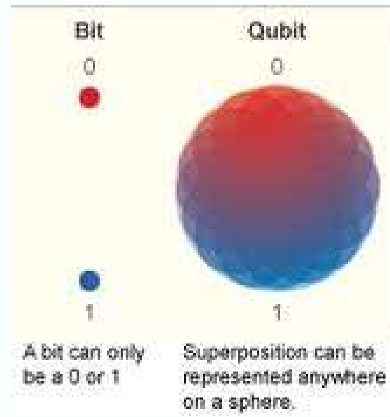
- Apply external B-field to project onto either one of two ferro states
- Measure magnetisation (m) as read-out

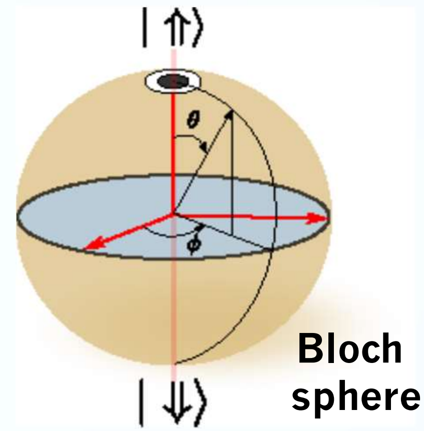
Powering up the Quantum Computer

- Each spin-1/2 is a 2-level system called a **qubit** living on it's own Bloch sphere.
- Instead of working with a toy model system of 2 spin-1/2s, we would like to work with a system of N interacting spin-1/2s (qubits).
- Enjoy the power of large-scale parallelisation of the system: any state of a system of N interacting spins will be a highly entangled state of many different classical states (the "outcomes").
- Processing of calculations via unitary evolution of quantum state keeps track of many different outcomes at once.

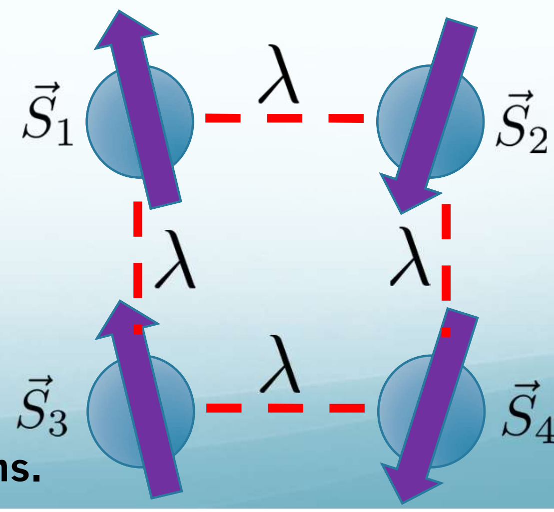
Note: Shor's quantum Fourier transform algorithm offers an exponential speedup compared to the best classical algorigthms.

Source: The internet





$$|\psi\rangle = \cos(\theta/2)|\uparrow\rangle + \sin(\theta/2)e^{i\phi}|\downarrow\rangle$$



Source: The internet

Sundar Pichai with Google's Sycamore Quantum Computer

What I work on.

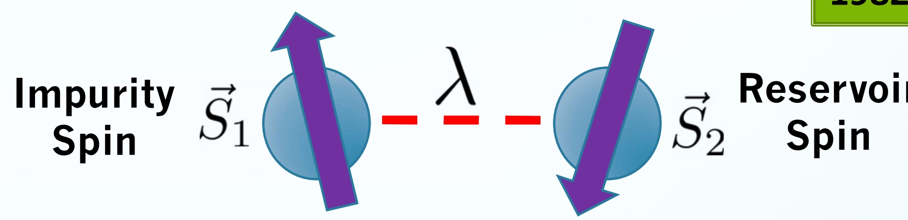
- EPQM's research involves building theoretical tools for the analysis & simulation of interacting many-particle quantum systems.
- These tools are equivalent to algorithms for quantum computation.
- Ground states we work with are heavily entangled, and large systems are very difficult to simulate on classical computers.
- Our research has yielded fresh, novel & exciting insights into several difficult problems of strongly interacting many-particle systems.

Source: The internet

Kondo

Wilson

1982



Kondo effect: impurity spin-1/2 coupled to spin-1/2 generated dynamically by reservoir of conduction electrons by Heisenberg interaction. Many-body version of 2-spin toy model!

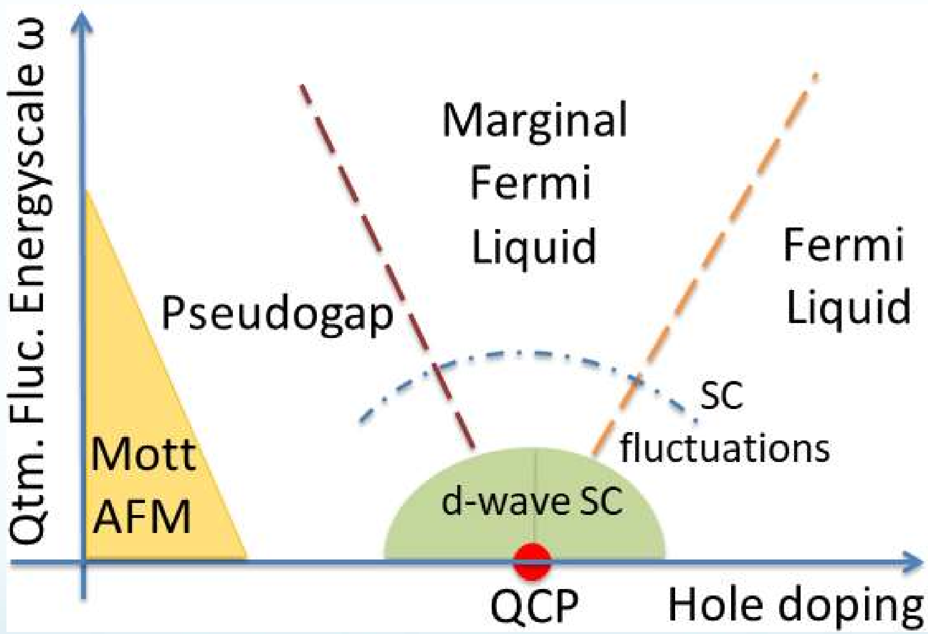
(See Mukherjee et al., PRB 105, 085119 (2022))

Another example: high-temperature superconductivity

Mott

❖ Nevil Mott proposed in the 50s that strong electronic repulsion can drive a system of interacting electrons into an insulating state. Several aspects of the Mott insulator remain mysterious even today.

We found the physics that leads to this phase diagram by developing methods that are quantum computation algorithms for the simulation of interacting systems of electrons.
(See, e.g., Mukherjee & Lal, JPCM 34, 275601 (2022))



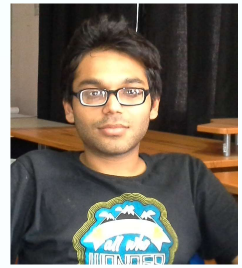
Experiments are performed at finite temperature (i.e., T is on the y-axis). Our analysis is at T=0; we have an energyscale for quantum fluctuations instead.

Anderson

❖ We develop methods by which to understand the puzzles posed here. The main challenge: can our answers lead to the design of a room-temperature superconductor?

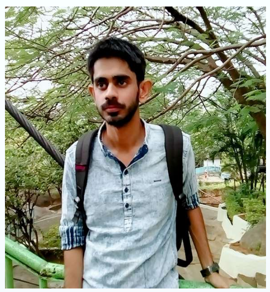
We are EPQM

Santanu Pal
Graduated in 2020



Anirban Mukherjee
Graduated in 2021

Siddhartha Lal
Graduated in 2003



Abhirup Mukherjee
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Siddhartha Patra
Graduated in 2022

Debraj Debata
Graduate Student (JRF)

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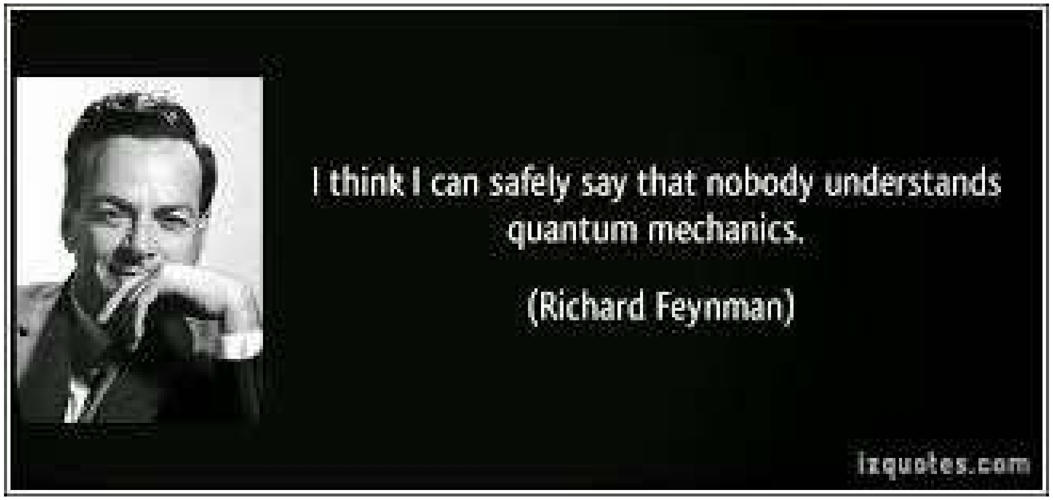
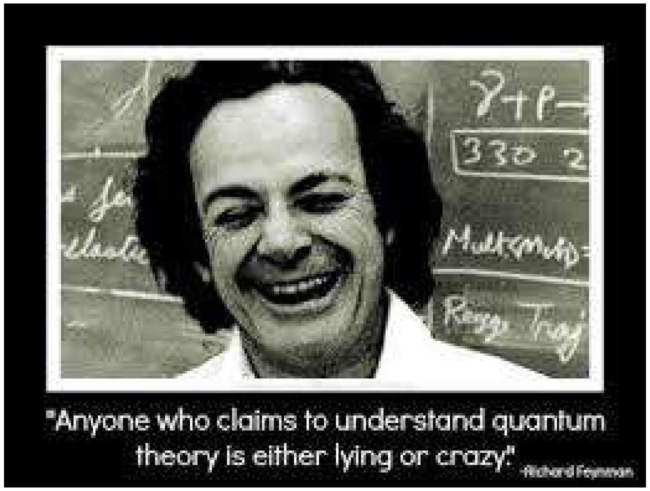
It's ok if you're lost, confused, excited ...

Source: The internet



Source: The internet



Nevertheless, we are building Quantum Computers today! And trying to solve difficult challenges using them.

Your excitement, enthusiasm & participation is invaluable.

Thank You!

E P Q M

- ✓ We have an opening for a graduate student/ Ph.D research scholar in our group starting from 2023. Interested I.Ph.D. students can contact me by email to learn more.

- ✓ BS-MS students interested in a MS project can email me to learn more.

email: slal@iiserkol.ac.in url: www.iiserkol.ac.in/~slal

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SERB INDIA

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Department of Physical Sciences